Math 542, Fall 2019

Computational Ordinary Differential Equations (CODEs)

Schedule Number: 22367

Instructor: Dr. Bo-Wen Shen

email: sdsu.math542.shen@gmail.com
URL: http://bwshen.sdsu.edu

(Last Updated: 2019/08/29)

COURSE INFORMATION

Class Days: TTh. (Tue. and Thur.)
Class Times: 11:00-12:15
Class Location: EBA-251

Dr. Bo-Wen Shen:
Office Hours Times (and by appointment): 2:30-3:30 pm TTH
Office Hours Location: GMCS 569

Course Overview

Course Descriptions:

Initial and boundary value problems for ordinary differential equations. Runge-Kutta, linear multi-step, predictor-corrector, adaptive, hybrid, shooting, and general linear methods. System, stiffness, and non-linear problems. Iterative methods.

Course Contents:

- The Euler Method and Runge-Kutta (RK) Method
- Multistep Methods: Adams-Bashforth (AB), Adams- Moulton (AM), Backward Differentiation Formula (BDF)
- Stability Analysis
- Stiff ODEs (e.g., boundary layer problems)
- Adaptive Methods (Variable step-size methods)
- Numerical Methods for BVPs, shooting method, finite difference methods
- Case studies, Lotka-Volterra Model, Lorenz model, the Van der Pol equation
- Python^, Symbolic Computing and Parallel Computing
- Galerkin Methods, Spectral and Finite Element (FE) Methods
- Numerical Packages and Solvers
- Finite Volume (FV) Methods

^Examples in the Lectures will be discussed mainly using Matlab or Python. For homework, students may choose their preferred computing language(s) such as C/C++, Fortran, Java and high-level script languages (e.g., Python, Matlab, R, or Julia).

Student Learning Outcomes:

- Outcome 1: Analyze various numerical methods and solvers and apply them to solve problems with ODEs. The methods include the Euler (forward/backward/improved) schemes, multi-stage Runge-Kutta methods, multi-step methods (Adams-Bashforth (AB), Adams- Moulton (AM), Backward differentiation formula (BDF)), and hybrid methods (e.g., predictor-corrector methods).

  Course Activity: Lectures.
  Assessment Strategy: Homework; midterm exam.

- Outcome 2: Derive the formula for numerical approximations to differential equations and perform stability analysis.

  Course Activity: Lectures.

Draft

Assessment Strategy: Homework.

- Outcome 3: Analyze numerical methods for high-order ODEs, PDEs and develop computer programming skills further. The following will be discussed for solving boundary value problems: (i) the shooting method; (ii) finite difference method; (iii) Rayleigh-Ritz method. Finite difference methods for solving PDEs are introduced as well as Galerkin Methods for Spectral and Finite Element methods (FEMs). A modern PDEtool package from Matlab will be introduced to solving PDEs using FEMs.

  Course Activity: Lectures.
  Assessment Strategy: Homework.

- Outcome 4: Apply the numerical methods introduced during the course for solving real-life problems.

  Course Activity: Lectures with case studies.
  Assessment Strategy: Homework.

- Outcome 5: Write a project proposal that applies the learned numerical methods to solving real-life problems; carry out the project by completing the proposed tasks and presenting results.

  Course Activity: Project-oriented problem in Homework
  Assessment Strategy: Oral presentation.

- Relation to Other Courses:

  Students may further take advanced courses such as partial differential equations (PDEs, Math531), high-performance computing and scientific computing (e.g., Math596), nonlinear dynamics (e.g., Math638), applied Fourier analysis (Math668), computational PDEs (e.g., Math693A,B).

Enrollment Information

Prerequisites: Math 337 (ODEs) and Math541 (Introduction to Numerical Analysis and Computing) with minimum grade of C. Understanding and knowledge in one of the following programing languages: C/C++, Fortran, Java, Python, Matlab, R, or Julia. Students with special requests may take this course subject to approval by the instructor of the course.

Course Materials

- Butcher, J. C., 2016: *Numerical Methods for Ordinary Differential Equations.* (3rd Edition), John Wiely & Sons, ISBN-13: 978-1119121503; ISBN-10: 1119121507.
- Boyce, W. E. and R. C. Diprima, 2012: *Elementary Differential Equations, Tenth Edition.* Wiely. ISBN-13: 978-0470458327 ISBN-10: 0470458321.
- Langtangen, H. P., 2016: *A Primer on Scientific Programming with Python.* Springer-Verlag Berlin Heidelberg. 914pp.

Course Structure and Conduct

Style of the Course: (1) Lectures/Discussions with power point slides and pdf materials; (2) practices on personal computers or a Linux cluster. (One user account per student is available upon request).

Course Assessment and Grading

- Homework 35%
- Mid Term Exam 30%
- Term Project 35%
- Attendance and discussions 2%

- Class Attendance: Students are required to attend all class meetings including project presentation and discussions. Class attendance will be taken randomly.
- Make-up exams: NO

Draft

Course Grade

You will be guaranteed the following grades as given by your percentage score on the homework, midterm, and final project presentation and report.

A 90%	B 80%	C 70%	D 60%
A- [89%, 90%)	B+ [85%, 89%)	C+ [75%, 79%)	D+ [65%, 69%)
	B [80%, 85%)	C [70%, 75%)	D [60%, 65%)
	B- [79%, 80%)	C- [69%, 70%)	D- [59%, 60%)

Important Dates

- August 26 (M), First day of Classes
- August 27 (T): First day of Class Math 542
- September 2 (M): Labor Day
- September 9 (M): Last day to officially withdraw from the university without penalty fee for Fall semester 2019.
- October 1 (T): Mid term (as discussed in the slides of the first lecture)
- October 3 (Th): A lecture for the preparation of projects
- October 8 (T): One paragraph of project descriptions
- October 17 (Th): Initial QuadChart
- November 2 (Sat): Last day to officially withdraw from all classes for Fall 2019 and receive a prorated refund (withdrawal after September 11 requires special approval and a penalty fee is assessed).
- November 11 (M): Veteran Day. Faculty/Staff Holiday
- November 19 (T); An updated QuadChart due
- November 28-29: Holiday—Thanksgiving recess
- December 11 (W): Last day of classes before final examinations
- December 12 (Th): Project presentations Part I
- December 17 (T): Project presentations Part II
- Dec 31 (T): Grades due from instructors. (11 p.m. deadline.)

Draft

Students with Disabilities

If you are a student with a disability and believe you will need accommodations for this class, it is your responsibility to contact Student Disability Services at (619) 594-6473. To avoid any delay in the receipt of your accommodations, you should contact Student Disability Services as soon as possible. Please note that accommodations are not retroactive, and that accommodations based upon disability cannot be provided until you have presented your instructor with an accommodation letter from Student Disability Services. Your cooperation is appreciated.

Academic Honesty

The University adheres to a strict policy regarding cheating and plagiarism. These activities will not be tolerated in this class. Become familiar with the policy (http://www.sa.sdsu.edu/srr/conduct1.html). Any cheating or plagiarism will result in failing this class and a disciplinary review by Student Affairs.

Examples of Plagiarism include but are not limited to:

- Using sources verbatim or paraphrasing without giving proper attribution (this can include phrases, sentences, paragraphs and/or pages of work)
- Copying and pasting work from an online or offline source directly and calling it your own
- Using information you find from an online or offline source without giving the author credit
- Replacing words or phrases from another source and inserting your own words or phrases
- Submitting a piece of work you did for one class to another class

If you have questions on what is plagiarism, please consult the policy (http://www.sa.sdsu.edu/srr/conduct1.html) and this helpful guide from the Library:(http://infodome.sdsu.edu/infolit/exploratorium/Standard_5/plagiarism.pdf)

At the end of the course, students will be ranked from first to last for each category (i.e. worst to be.st). The number of participation points allocated will then be based on each student's overall relative ranking score.

Interacting with me

I'll try to respond within 24-48 hours to emails sent me (sdsu.math542.shen@gmail.com) from within Blackboard. For quick questions, the turnaround time may be much shorter. (If Blackboard is not working or for non-course-related communications, write to me using my work email address: bshen@mail.sdsu.edu.)

Please feel free to call me in my office at (619) 594-5962. My message phone works most of the time, but email is to be preferred. My regular office hours are TTH 3:00-4:00pm Pacific Time.

Course Outline

Chapter	Sections	Remarks
A: Basic (IVPs)		
	A0. Overview & Preparation (e.g., QuadChart)	
	A1. The Euler Method (forward vs. backward) & Error Analysis	
	A2. Improvement on the Euler Method (the Heun Formula)	~0.5 lecture
	A3. The Runge-Kutta (RK)Method	
	A4. An Introduction to Python (symbolic computing, plotting)	Programming
	A5. Multistep Methods (e.g., Adams-Bashforth, Adams-Moulton)	
	A6. More on Errors; Stability	
B: Advanced (IVPs)		
	B0. The Elementary Theory of IVPs and Lipschitz Condition	0.5 lectures
	B1. More on Runge-Kutta (RK) Methods and Stability Analysis	2 lectures
	B2. Application of RK Methods to Lotka-Volterra Models	A case study
	B3. More on Linear Multistep Methods	2 lectures
	B4. The Lorenz system and its high-dimensional versions	A case study
	B5. Predictor-Corrector Methods	
	B6. Stiff ODEs -- Multiscale Phenomena	
	B7. Runge-Kutta Methods for Stiff ODEs	
	B8. Linear Multistep Methods for Stiff Systems	
	B9. Example: Variable Order LMM Schemes Example: The Van der Pol Equation	Case studies
	B10. Example: Adaptive RKF45 Solver	
	B11. Hybrid Methods	
	B12. General Linear Methods	
	B13. ODE Solvers and Their Applications	*, Programming
	B14. Parallel computing	*, Programming
C: BVPs		
	C1. Boundary Value Problems (BVPs)– Introduction	
	C2. BVPs: The Shooting Method	
	C3. Finite Difference Methods	
	C4. Higher Order Equations	*
	C5. Nonlinear Equations	*

Draft

D: Selective*		
	D1. Linearization and Eigenvalue Analysis (Stability of multi-dimensional flows)	0.5 lectures
	D2. Numerical Calculations of Lyapunov Exponents	*, 0.5 lectures
	D3. Fourier Transform and Sturm-Liouville Problem	*
	D4. Galerkin Methods (Spectral and Finite Element)	
	D5. Finite-Volume methods	*, 1.5 lectures
Project Presentations by Students		
	The goal for this course is to encourage students to learn numerical methods and apply them to solving problems for their research projects (including thesis). Innovative approaches and/or methodologies by applying existing schemes and packages are also encouraged.	a 10-12 minutes presentation per student

*Optional: these lectures will be presented subject to time availability.

Draft